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Impact Ionization in Crossed Fields in Semiconductors

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Starting from the motion of the free electron, a model is developed for the impact ionization of electron-hole pairs in the presence of a strong transverse magnetic field. The ionization rate was found to depend only on the quotient of the effective electric field strength divided by the magnetic field E_{eff}/B. For n-InSb the ionization rate was calculated in dependence of E_{eff}/B for different values of the mean free path for optical-phonon scattering. Impact ionization begins at $E_{\text{eff}}/B = 2.7\times10^5$ m sec^{-1}.

I. INTRODUCTION

The problem of impact ionization in semiconductors in crossed electric and magnetic fields was first investigated by Toda and Glicksman.[1] They suggested the "transverse breakdown," and impact ionization in the Hall direction at comparable or lower electric fields than in the absence of a magnetic field. But measurements by Ferry and Heinrich[2] showed a decrease of the ionization rate caused by the magnetic field. This result was in agreement with unpublished measurements done in our laboratory. Therefore, and because of our interest in the ionization process in the somewhat different geometric conditions of a θ pinch,[3] we developed a model for the impact ionization in crossed fields, in which the Hall field is regarded as an independent physical quantity. The decrease of the ionization rate predicted by our model therefore does not exclude the existence of a transverse breakdown, but only indicates that this effect will be caused by a nonlinear dependence of the Hall field strength on the magnetic field, as was also predicted by Schmidt and Nelson.[4]

Our model is valid for a semiconductor with the following properties: (a) moderately doped n type, (b) parabolic conduction band, (c) dominant optical-phonon scattering, (d) isotropic effective mass. Starting from the equation of motion of a free electron, we investigated (Secs. II–V) the behavior of an electron in a magnetic field. The velocity vector of such an electron is moving on a gyration circle which is not centered in the origin of the velocity plane. To get a one-dimensional (energy) description, we divided this gyration circle into a "fast" half and a "slow" one, as determined by the absolute value of the velocity. The energy of the electron was averaged separately over each half circle and the continuous motion of the velocity vector on the gyration circle was replaced by transitions between the two energy mean values. With these transitions and those caused by the scattering with acoustical and optical phonons, the Boltzmann equations and two continuity equations for the motion of the particles in the energy space are formulated (Sec. VI). In Sec. VII this system of equations is solved and the energy distribution of the electrons is determined. The ionization rate, the number of electron-hole pairs produced by one electron within unit time, is given in Sec. VIII. It was found to depend on $u = (E_z^2 + E_H^2)^{1/2}/B$ only. Here E_z is the external electric field, E_H is the Hall field, and B is the magnetic field.

II. SOLUTION OF EQUATION OF MOTION

Moving in the crystal the electrons are not subject to a homogeneous friction, but lose their energy by single impacts. Between the impacts the behavior of the electrons is described by the equation of motion for free particles:

$$\frac{d\vec{v}}{dt} = -\frac{e}{m^*}(\vec{E} + \vec{v}\times\vec{B}), \tag{1}$$

where t is the time, $-e$ is the electric charge of the electron, m^* is the effective mass, $\vec{v}$ is the velocity, $\vec{B}$ is the magnetic field, and $\vec{E}$ is the electric field at the position of the particle. The electric field is the sum of the applied external electric field E_z and the Hall field E_H.

If the magnetic field is perpendicular to the elec-

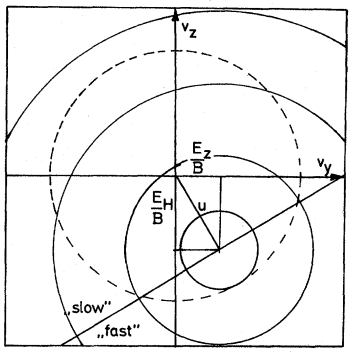

FIG. 1. Gyration circles (solid circles) in the z-y plane of the velocity space, circle of constant energy (dashed), and boundary between the slow and the fast half-circles.

tric fields, as is always assumed here, and if we choose the following spatial arrangement in Cartesian coordinates:

$$\vec{E}=\begin{pmatrix}0\\E_H\\E_z\end{pmatrix} \text{ and } \vec{B}=\begin{pmatrix}B\\0\\0\end{pmatrix},$$

then no force on the electrons in the x direction will exist. It is thus possible to change from vectors to complex coordinates:

$$v=v_z+iv_y,$$
$$\frac{dv}{dt}=-\frac{e}{m^*}\,(E_z+iE_H+ivB). \tag{2}$$

The solution of this equation is given by

$$v(t)=\left(v_0+\frac{E_H-iE_z}{B}\right)e^{-i\omega_c t}-\frac{E_H-iE_z}{B}, \tag{3}$$

where $\omega_c=eB/m^*$ is the gyration frequency and $v_0=v_{0z}+iv_{0y}$ is the velocity of the particle at the time $t=0$. The first term of $v(t)$ represents concentric circles in the z-y plane of the velocity space if E_z, E_H, and B are constant. Their radii

$$R=\left[\left(v_{0z}+\frac{E_H}{B}\right)^2+\left(v_{0y}-\frac{E_z}{B}\right)^2\right]^{1/2} \tag{4}$$

depend on the initial conditions only. The second term in (3) displaces the center of the gyration circles by the constant drift velocity $(iE_z-E_H)/B$ out of the origin (Fig. 1). This means that between impacts the electrons run on closed orbits within the velocity space in a certain and, for most of them, rather narrow region of energy. They change quickly between the minimal and the maximal energy of their orbit. But to increase their energy above the maximal energy of the orbit, they have to be scattered onto a circle with a larger radius.

In order to get a condition for the energy gain, we divide the velocity plane by a line into two parts so that each gyration circle is divided into a "slow" and a "fast" half. Fast means that everywhere on this half-circle the kinetic energy of a particle is higher than anywhere on the slow one. Disregarding at first the loss of energy by phonons, one realizes an increase of the radius and also the possibility of an increase of the energy in the case that the electron undergoes an impact and is scattered from the fast into the slow half of the velocity plane. Scattering into the opposite direction results in a loss of radius and therewith the possibility of a loss of energy.

III. DESCRIPTION OF MODEL

The approximation which is characteristic for our model is the replacing of the continuous change of energy under the influence of the fields while the electrons are moving on the gyration circles by a jumping between the two average values of the energy of the slow and the fast half-circles.

Let W be the kinetic energy, φ be the phase angle of a particle on the gyration circle, and $p(\varphi)$ be the probability for the particle to have the phase φ, then the time-averaged value of the energy W is given by

$$\bar{W}=\int_{\varphi_1}^{\varphi_2}W(\varphi)p(\varphi)d\varphi\Big/\int_{\varphi_1}^{\varphi_2}p(\varphi)d\varphi\,. \tag{5}$$

$p(\varphi)$ certainly is a function of τ, the mean time between two scattering processes. $\omega_c\tau$ gives the average part of gyration for the electrons between two impacts. The larger $\omega_c\tau$, the less $p(\varphi)$ will vary with φ. Especially if $\omega_c\tau$ is in the range of 2π or larger, $p(\varphi)$ will be nearly constant. Then (5) is simplified to

$$\bar{W}=\frac{1}{\varphi_2-\varphi_1}\int_{\varphi_1}^{\varphi_2}W(\varphi)d\varphi \quad \text{when} \quad \omega_c\tau\geq 2\pi. \tag{6}$$

$\omega_c\tau\geq 2\pi$ is a condition for the magnetic field:

$$B\geq 2\pi m^*/e\tau, \tag{7}$$

which means that unfortunately the model developed here cannot be applied for the interesting limit $B\to 0$.

IV. CALCULATION OF AVERAGE VALUES OF ENERGY

From Eq. (3) one obtains the kinetic energy of the particles:

$$W=\tfrac{1}{2}m^*|v|^2=m^*\left[\frac{E_H^2+E_z^2}{B^2}+v_{0z}\frac{E_H}{B}-\frac{E_z}{B}v_{0y}\right.$$
$$+\tfrac{1}{2}(v_{0z}^2+v_{0y}^2)+\left(v_{0y}\frac{E_z}{B}-v_{0z}\frac{E_H}{B}-\frac{E_H^2+E_z^2}{B^2}\right)\cos\omega_c t$$
$$\left.-\left(v_{0z}\frac{E_z}{B}+v_{0y}\frac{E_H}{B}\right)\sin\omega_c t\right]\,. \tag{8}$$

To calculate the average values, let us consider an electron which has its minimum energy at the time $t=0$. Then the slow half-circle extends from

$\varphi = \omega_c t = -\frac{1}{2}\pi$ to $\frac{1}{2}\pi$ and the fast one from $\frac{1}{2}\pi$ to $\frac{3}{2}\pi$. Moreover, the choice of the time origin gives a relation between v_{0x} and v_{0y}:

$$v_{0x}/v_{0y} = -E_H/E_x \ . \tag{9}$$

With (4) one obtains

$$v_{0x} = \frac{E_H}{(E_H^2+E_x^2)^{1/2}}\ R - \frac{E_H}{B} \tag{10}$$

and

$$v_{0y} = \frac{-E_x}{(E_H^2+E_x^2)^{1/2}}\ R + \frac{E_x}{B} \ . \tag{11}$$

Thus Eq. (8) becomes

$$W(R,t) = \frac{m^*}{2}\left[R^2 + \frac{E_H^2+E_x^2}{B^2} - \left(\frac{E_H^2+E_x^2}{B^2}\right)^{1/2} R\cos\omega_c t\right]. \tag{12}$$

In (12) the electric and magnetic fields appear explicitly only in the combination $(E_x^2+E_H^2)/B^2$, that is, the square of the drift velocity u of the gyration center. With $u = (E_H^2+E_x^2)^{1/2}/B$

$$W(R,t) = \tfrac{1}{2}m^*(R^2+u^2-u\,R\cos\omega_c t). \tag{13}$$

This leads to the average values

$$W_s = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} W\, d\omega_c t = \frac{m^*}{2}\left[\left(1-\frac{4}{\pi^2}\right)u^2 + \left(R-\frac{2}{\pi}u\right)^2\right] \tag{14}$$

for the slow half and

$$W_f = \frac{1}{\pi}\int_{\pi/2}^{3\pi/2} W\, d\omega_c t = \frac{m^*}{2}\left[\left(1-\frac{4}{\pi^2}\right)u^2 + \left(R+\frac{2}{\pi}u\right)^2\right] \tag{15}$$

for the fast half of the gyration circle.

Figure 2 shows W_s and W_f as functions of the radius R. They are two parabolas which intersect at the W axis. The minimum values for W are larger than zero because the state $W=0$ cannot be considered isolated any more due to the averaging process.

V. TRANSITIONS BY SCATTERING

We are interested in the impact processes not only with regard to the energy loss, but also in order to get the direction in which the particle is scattered after the collision. The loss or gain of the radius depends on this direction. The averaging of the energy is the reason that the state of motion of the particle is no longer characterized by its velocity vector, but by its averaged energy and the index s or f corresponding to the region of the velocity space in which it stays. The question of the direction then becomes only a question of whether the index s or f changes after the collision or not. This means we have only to distinguish between forward and backward scattering in the same way as

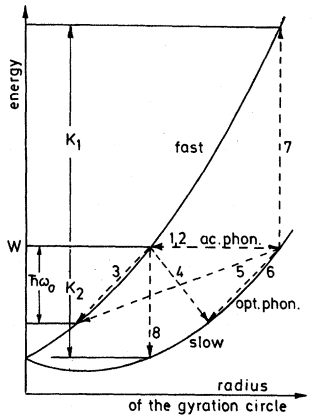

FIG. 2. Mean values of the energy of the fast and the slow half-circles plotted as a function of the radii R of the gyration circles and the possible transitions from a given energy value W.

was done by Dumke.[5] The scattering times are characterized by the indices o and a for optical and acoustical-phonon scattering and v and b for forward and backward scattering:

$$\frac{1}{\tau_o} = \frac{1}{\tau_{ov}} + \frac{1}{\tau_{ob}} \quad \text{and} \quad \frac{1}{\tau_a} = \frac{1}{\tau_{av}} + \frac{1}{\tau_{ab}} \ .$$

From the two states of motion belonging to one value of energy the following transitions are possible (Fig. 2).

(a) The electrons can undergo collisions by which acoustical phonons are emitted. The loss of energy is negligibly small and therefore forward scattering does not change the state of motion. There are only the transitions 1 and 2 in Fig. 2 by which the backward-scattered electrons pass over to the other state of motion belonging to the same energy.

(b) By optical-phonon scattering the particles lose the constant energy $\hbar\omega_0$. Therefore by forward and backward scattering they reach the two states of motion, the energy of which is lower by $\hbar\omega_0$ (transitions 3 to 6).

(c) Two further transitions describe the behavior of the electrons under the influence of the electric and magnetic fields. These are vertical transitions (7 and 8) in Fig. 2, corresponding to the change of the particle into the other half of its gyration circle. The frequency of these transitions is ω_c/π. The energy gain or loss can be calculated from Eqs. (14) and (15). We obtain for transition 7

$$K_1 = \frac{8m^*}{\pi^2}u^2 \pm \frac{4m^*}{\pi}u\left[\frac{2W}{m^*} - \left(1-\frac{1}{\pi^2}\right)u^2\right]^{1/2} \tag{16}$$

and for transition 8

$$K_2 = \frac{8m^*}{\pi^2}u^2 - \frac{4m^*}{\pi}u\left[\frac{2W}{m^*} - \left(1-\frac{4}{\pi^2}\right)u^2\right]^{1/2}, \tag{17}$$

where K_2 is a negative quantity. The ambiguity of

the sign of the second term of K_1 results from an ambiguity of W_s for $W_s \leq \frac{1}{2}mu^2$. The following calculation involves the region $W > \frac{1}{2}mu^2$ only, where the positive sign is valid.

(d) Above the ionization energy W_i, ionization impacts also occur in which the particles lose all of their energy generating a new electron-hole pair.

VI. FOUR BASIC EQUATIONS

The transitions mentioned in Sec. V completely determine the behavior of the electrons. It is possible to formulate with them the Boltzmann equation for the distribution function $s(W)$ of the particles staying just in the slow half of their gyration circle, and for the distribution function $f(W)$ for the electrons in the fast half. The sum of these two functions gives the energy distribution for all electrons $n(W)$:

$$n(W) = s(W) + f(W).$$

The Boltzmann equations have the form

$$\frac{\partial s}{\partial t} + \frac{\partial s}{\partial W}\frac{\partial W}{\partial t} = \left.\frac{Ds}{Dt}\right|_{\text{transitions}} \tag{18}$$

and

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial W}\frac{\partial W}{\partial t} = \left.\frac{Df}{Dt}\right|_{\text{transitions}} . \tag{19}$$

The second terms on the left-hand sides vanish because the continuous change of energy is replaced by energy jumps, all of which are included in the terms on the right-hand sides. In the stationary state the partial derivatives with respect to the time also vanish, and therefore the left-hand sides are zero:

$$0 = \left.\frac{Ds}{Dt}\right|_{\text{transitions}} = -s\left(\frac{1}{\tau_o} + \frac{1}{\tau_{ab}} + \frac{\omega_c}{\pi}\right) + \frac{f}{\tau_{ab}} + \int_0^\infty dW' \left[\delta(W+\hbar\omega_0 - W')\left(\frac{f(W')}{\tau_{ob}(W')} + \frac{s(W')}{\tau_{ov}(W')}\right) + \delta(W+K_1-W')\frac{\omega_c}{\pi} f(W')\right]$$

and

$$0 = \left.\frac{Df}{Dt}\right|_{\text{transitions}} = -f\left(\frac{1}{\tau_o} + \frac{1}{\tau_{ab}} + \frac{\omega_c}{\pi}\right) + \frac{s}{\tau_{ab}} + \int_0^\infty dW' \left[\delta(W+\hbar\omega_0 - W')\left(\frac{s(W')}{\tau_{ob}(W')} + \frac{f(W')}{\tau_{ov}(W')}\right) + \delta(W+K_2-W')\frac{\omega_c}{\pi} s(W')\right].$$

The integrals describe the growth of the distribution function resulting from particles coming from other energy values. After calculation of the integrals one obtains

$$0 = -s\left(\frac{1}{\tau_o} + \frac{1}{\tau_{ab}} + \frac{\omega_c}{\pi}\right) + \frac{f}{\tau_{ab}} + \frac{s(W+\hbar\omega_0)}{\tau_{ov}(W+\hbar\omega_0)} + \frac{f(W+\hbar\omega_0)}{\tau_{ob}(W+\hbar\omega_0)} + \frac{\omega_c}{\pi} f(W+K_1) \tag{20}$$

and

$$0 = -f\left(\frac{1}{\tau_o} + \frac{1}{\tau_{ab}} + \frac{\omega_c}{\pi}\right) + \frac{s}{\tau_{ab}} + \frac{f(W+\hbar\omega_0)}{\tau_{ov}(W+\hbar\omega_0)} + \frac{s(W+\hbar\omega_0)}{\tau_{ob}(W+\hbar\omega_0)} + \frac{\omega_c}{\pi} s(W+K_2). \tag{21}$$

In the following, two other equations for the flux of particles through the energy distribution are derived. We assume that the influence of the recombination and the newly generated particles on the energy distribution is negligibly weak, and furthermore that the electrons lose their whole energy in the ionization impacts which means that they reenter the distribution at energy zero. Then the conservation of the stationary state requires a flux of particles through the distribution which compensates for the loss of electrons with energies above the ionization energy due to pair generation. This flux divided by the total number of electrons contained in the distribution gives the ionization rate.

Below the ionization energy the flux of particles, Φ, is constant. We are able to set up two equations for this: one for the flux reaching an energy value from below and another for the flux leaving the energy value in the upward direction. Because there are different transitions from each, these are given by the sums of the fluxes belonging to the different transitions;

$$\Phi = \frac{\omega_c}{\pi} K_1 s(W) - \frac{\omega_c}{\pi} K_1 f(W+K_1) - \hbar\omega_0 \frac{n(W+\hbar\omega_0)}{\tau_o(W+\hbar\omega_0)} \tag{22}$$

is the equation for the flux leaving an energy value in the upward direction. The first term on the right-hand side describes the particles which change from the slow into the fast half and gain thereby the energy K_1, while the second describes the opposite process, i.e., particles coming back from the fast into the slow half losing the energy K_1. The last term stands for the electrons falling down from higher to the considered energy due to optical-phonon scattering.

The equation for the flux from below is

$$\Phi = \frac{\omega_c}{\pi} K_2 f(W) - \frac{\omega_c}{\pi} K_2 s(W+K_2) - \hbar\omega_0 \frac{n(W)}{\tau_o(W)} . \tag{23}$$

Here again the first two terms mean the gyration and the last one the optical-phonon scattering (K_2 is negative). These two equations, (22) and (23), and the Boltzmann equations (20) and (21) form the basic

system which allows us to determine the distribution function $n(W)$ and the ionization rate g.

VII. SOLUTION OF SYSTEM OF EQUATIONS

Addition of Eqs. (20) and (21) and elimination of $s(W+K_2)$ and $f(W+K_1)$ using the two equations for Φ gives

$$\Phi\left(\frac{1}{K_1}+\frac{1}{K_2}\right)=-\left(1+\frac{\hbar\omega_0}{K_2}\right)\frac{n(W)}{\tau_o(W)} + \left(1-\frac{\hbar\omega_0}{K_1}\right)\frac{n(W+\hbar\omega_0)}{\tau_o(W+\hbar\omega_0)} \quad . \quad (24)$$

We use the following approximation:

$$\frac{h(W+\hbar\omega_0)}{\tau_o(W+\hbar\omega_0)} \approx \frac{n(W)}{\tau_o(W)} + \hbar\omega_0\left[n(W)\frac{\partial}{\partial W}\left(\frac{1}{\tau_o}\right)+\frac{1}{\tau_o(W)}\frac{\partial n}{\partial W}\right]. \quad (25)$$

Under the assumption that the mean free path for optical-phonon scattering is constant, [6] i.e., that $1/\tau_o$ is proportional to $W^{1/2}$, one obtains

$$\tau_o\frac{\partial}{\partial W}\left(\frac{1}{\tau_o}\right)=\frac{1}{2W} \ ,$$

$$\Phi=-\left(1+\frac{K_2(\hbar\omega_0-K_1)}{2W(K_1+K_2)}\right)\frac{\hbar\omega_0}{\tau_o}n - \frac{\hbar\omega_0-K_1}{K_1+K_2}K_2\frac{\hbar\omega_0}{\tau_o}\frac{\partial n}{\partial W} \quad . \quad (26)$$

In Eq. (26) the electric and magnetic fields appear only through K_1 and K_2 in the combination $u=(E_H^2+E_E^2)^{1/2}/B$, the drift velocity of the gyration center, and the relaxation time for acoustical-phonon scattering as well as the distinction between forward and backward scattering with optical phonons do not appear.

Below the ionization energy Φ is constant and may be denoted by $\Phi_0=\Phi(W_i)$. Above the ionization energy Φ decreases because of the particles which have made ionization impacts below the energy considered. That means

$$\Phi(W)+\int_{W_i}^{W}\frac{n(W)}{\tau_i}\,dW=\Phi_0, \quad (27)$$

where τ_i is the mean time for ionization impacts. This equation leads to two others:

$$\frac{\partial\Phi}{\partial W}=-\frac{n(W)}{\tau_i} \quad (28)$$

and

$$\int_{W_i}^{\infty}\frac{n(W)}{\tau_i}\,dW=\Phi_0 \ . \quad (29)$$

Using (28) in (26) one obtains a second-order differential equation for the distribution function $n(W)$ in the high-energy region $W>W_i$:

$$K_2\left(\frac{\hbar\omega_0-K_1}{K_1+K_2}\right)\frac{\hbar\omega_0}{\tau_0}\frac{\partial^2 n}{\partial W^2}+\left[3-\frac{2\hbar\omega_0}{K_1-K_2}+\frac{K_2}{W}\left(\frac{\hbar\omega_0-K_1}{K_1+K_2}\right)\right]\frac{\hbar\omega_0}{\tau_0}\frac{\partial n}{\partial W} + \left\{\frac{\hbar\omega_0}{2W\tau_0}\left[3-\frac{2\hbar\omega_0}{K_1-K_2}-\frac{K_2}{2W}\left(\frac{\hbar\omega_0-K_1}{K_1+K_2}\right)\right]-\frac{1}{\tau_i}\right\}n=0. \quad (30)$$

It is possible to solve Eq. (30) analytically if constant coefficients are assumed. We will use for these coefficient values which correspond to the ionization energy. The error due to this approximation is certainly small, because above W_i the distribution function decreases quickly to zero, and therefore most of the particles of the high-energy region will be found near the ionization energy. By arbitrarily fixing the distribution function at the point $W=W_i$ as unity $[n(W_i)=1]$, we get

$$n(W)=e^{-\lambda W} \ , \quad (31)$$

with

$$\lambda=\frac{1}{2W_i}+\frac{[3-2\hbar\omega_0/(K_1-K_2)](K_1+K_2)}{2(\hbar\omega_0-K_1)K_2}+\left[\left(\frac{[3-2\hbar\omega_0/(K_1-K_2)](K_1+K_2)}{2(\hbar\omega_0-K_1)K_2}+\frac{1}{2W_i}\right)^2 - \frac{[3-2\hbar\omega_0/(K_1-K_2)](K_1+K_2)}{2W_i(\hbar\omega_0-K_1)K_2}+\frac{1}{4W_i^2}+\frac{(K_1+K_2)\tau_0/\tau_i}{(\hbar\omega_0-K_1)K_2\hbar\omega_0}\right]^{1/2} \ . \quad (32)$$

Using the obtained distribution function in this way for the high-energy region in Eq. (29), one can calculate Φ. To get the ionization rate g, the flux Φ has to be divided by the total number of electrons contained in the distribution. That requires the determination of the portion of the distribution function for $W<W_i$ from (26). The general solution of (26) is

$$n(W)=\exp\left[-\int_{W_0}^{W}\left(\frac{K_1+K_2}{K_2(\hbar\omega_0-K_1)}+\frac{1}{2W}\right)dW\right]\left\{1+\Phi\int_{W_0}^{W}\frac{\tau_0(K_1+K_2)}{\hbar\omega_0 K_2(\hbar\omega_0-K_1)}\exp\left[\int_{W_0}^{W}\left(\frac{K_1+K_2}{K_2(\hbar\omega_0-K_1)}+\frac{1}{2W}\right)dW\right]dW\right\}. \tag{33}$$

The integrals in the exponents can be solved

$$-\int_{W_0}^{W}\left(\frac{K_1+K_2}{K_2(\hbar\omega_0-K_1)}+\frac{1}{2W}\right)dW=\frac{\hbar\omega_0-\frac{1}{2}(K_1+K_2)}{K_1+K_2-\hbar\omega_0}\ln\left|\frac{\hbar\omega_0-K_1(W)}{\hbar\omega_0-K_1(W_0)}\right|$$
$$-\frac{1}{2}\frac{K_1+K_2}{K_1+K_2-\hbar\omega_0}\ln\left|\frac{K_2(W)}{K_2(W_0)}\right|-\frac{1}{2}\ln\left|\frac{W}{W_0}\right|. \tag{34}$$

The remaining integral in (33) was solved numerically. The solution was obtained by starting at W_i and going downward to the energy of the optical phonon $\hbar\omega_0$. In agreement with Dumke we assumed the distribution function to be constant below this energy since the most important mechanism for the loss of energy, the optical-phonon scattering, cannot occur there.

Integration of the now completely known distribution gives the total number of electrons N:

$$N=\int_0^\infty n(W)\,dW=\hbar\omega_0\,h(\hbar\omega_0)+\int_{\hbar\omega_0}^\infty n(W)\,dW. \tag{35}$$

Division of the flux of particles Φ by their total number N leads to the ionization rate

$$g=\Phi/N\ .$$

VIII. RESULTS AND DISCUSSION

A numerical calculation of the ionization rate was done for n-InSb. The following values were used:

$$W_i=0.25\text{ eV},\quad \hbar\omega_0=0.025\text{ eV},\quad \tau_i=2\times10^{-11}\text{ sec},$$
$$u=2.7\text{–}5.9\times10^5\text{ m sec}^{-1},\qquad T=77\,^\circ\text{K},$$

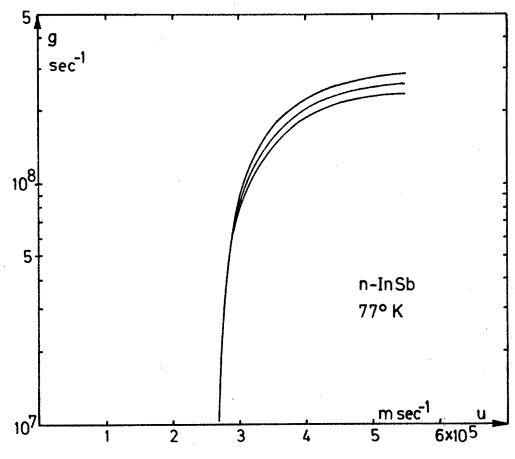

FIG. 3. Ionization rate of n-InSb (77 °K) plotted as a function of $u=E_{\text{eff}}/B$. Upper line $\lambda_{\text{opt}}=0.25\,\mu$, middle line $\lambda_{\text{opt}}=1.25\,\mu$, and lowest line $\lambda_{\text{opt}}=2.5\,\mu$.

$$\lambda_{\text{opt}}=0.25\ \mu,\ 1.25\ \mu,\ 2.5\ \mu.$$

The values for τ_i and the ionization energy W_i were taken from Dumke's paper. Since the mean free path for optical-phonon scattering is not known precisely, it was varied in order to determine its influence on the ionization rate.

As mentioned just before, the ionization rate depends on the electric and magnetic fields only through the quantity $u=|\vec{E}_{\text{eff}}|/|\vec{B}|$. This dependency shows the analogy of the problems treated here to those in gas plasma physics.[7] The numerical results are given in Fig. 3. Below $u=2.7\times10^5$ m/sec impact ionization does not occur, but above this value the ionization rate rises rapidly and shows a saturation at a value between 2×10^8 and 3×10^8 sec^{-1}. The influence of the mean free path on the ionization rate is remarkably small: Although λ_{opt} in the highest and in the lowest curve differ by a factor 10, all curves are inside the limits of error expected for such experiments. Furthermore, it is striking that the ionization rate increases with decreasing mean free path, whereas a decrease is obtained in the theories for impact ionization in an electric field alone.[6] The increase results from the fact that the energy loss due to optical phonons is less important here than the possibility of the particle being scattered into a gyration circle with a larger radius.

Finally in all the experiments the value of the ionization rate is influenced also by the Hall field, whose dependency on the applied electric and magnetic fields may be complicated in special cases. Therefore it was not possible to give a quantitative comparison of the theory with the experiments mentioned above. Certainly no formula exists which is valid for all geometries and all plasma densities. At this point further microscopic investigations should begin. They would also give information about the transverse breakdown observed by Toda and Glicksman, perhaps in the way, that under special conditions of the geometry and the plasma density, a region will exist in which the Hall field increases more than proportionally to the magnetic field.

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Statistical Mechanics of Charged Traps in an Amorphous Semiconductor*†

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The effect of a Coulomb interaction between charged traps in an amorphous semiconductor is investigated within the premises of the Mott-Cohen-Fritzsche-Ovshinsky model. The grand partition function is expressed as a functional integral over a set of Gaussian random fields. The free energy is expressed as a sum of the mean-field result plus fluctuations about the mean field. It is shown that for the system under consideration, the mean field is just the Hartree self-consistent field and that at $T=0$ °K it represents the exact ground state. It is shown that the fluctuations about the mean field represent correlations in the system. Approximate expressions for the mean occupation number and the renormalized energies of the charges are obtained as well as the renormalized single-particle density of states. The excitation spectrum of single quasiparticles, within any given band, is shown to have a quasigap. It is shown that the effect of a Coulomb interaction between the charged traps is to reduce the density of states at the Fermi energy by a factor of 2 below its value in the absence of interactions.

I. INTRODUCTION

The discovery of novel phenomena in amorphous semiconductors has prompted considerable theoretical interest in these materials. The theoretical attempts are aimed towards the understanding of the electronic structure and transport in disordered systems, in particular in covalent amorphous semiconducting alloys. There exists, as yet, no rigorous theory of the electronic structure in disordered systems. There has emerged, however, a basic band model (which was synthesized by Mott[1] out of earlier work) which illustrates the universal features of the electronic structure of disordered materials. This basic band model has been further elaborated and clarified, in the context of amorphous semiconductors, by Cohen, Fritzsche, and Ovshinsky[2] (we will refer to it as the Mott-CFO model). The basic features of their model are displayed in Fig. 1 via a sketch of the density of states. For our purpose it is necessary to briefly review some of the essential assumptions underlying the model. They postulate that in an amorphous semiconductor there exist bands of extended states, that these bands have tails of localized states, and that in sufficiently disordered materials (such as alloys or bad films) these tails overlap in the forbidden gap. They further assume that every localized state has a well-defined parentage, i.e., it is always possible to assign a valence or conduction character to a localized state in the gap. The overlapping of the tails and the fact that there is a finite density of localized states at the Fermi energy has many interesting consequences. Since valence states are electrically neutral when occupied and conduction states electrically neutral when empty, there results a random distribution of localized charged traps throughout the material, positive trapped holes (corresponding to the empty valence-band tail above the Fermi energy E_F) and negative trapped electrons (corresponding to the occupied conduction-band tail below E_F). The overall electrical neutrality of the material is guaranteed through a proper choice of E_F. One believes that there may be as many as 10^{19} of these localized states (per $\mathrm{cm}^3\,\mathrm{eV}$) at E_F. One anticipates that such a distribution of localized charges will have a significant effect on carrier kinetics. One would like to know, for example, what the effect would be of these localized charges on the carrier concentra-